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A Probabilistic Model for Link-11 Networking Operation

JOHN S. SCHLORFF

Communication Systems Branch
Information Technology Division

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John S. Schlorff

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In this report, we model the networking operation of the Link-11 tactical data network. The model is a Markov chain analysis of the finite state machine that controls the polling sequence in the Link-11 Data Net Control Station. We calculate the net cycle time, the effective throughput, and the channel utilization of the network based on modem performance metrics. The model is based on a number of simplifying assumptions, which include uniform channel conditions for all nodes and a uniform and simplified message generation function for all node. The model also assumes that no collisions in transmission can occur.

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CONTENTS

A PROBABILISTIC MODEL FOR LINK-11 NETWORKING OPERATION

1 Introduction

This report describes an analytical model used to evaluate the Link-11 tactical data network. The baseline description of the Link-11 networking protocols used in our model was obtained from reference [1], and is summarized in section 2 of this memorandum. The reader should already be familiar with the Link-11 system, its networking protocols, its waveform, and the Improved Link-11 (LEI) waveform (reference [2]). The purpose of the model is to provide the capability to compare the performance of the Link-11 network when the network is operating with different modems. The model can accommodate both the currently used parallel tone modems and the new proposed single tone modems. The performance measures calculated in this model are the net cycle time, the percent channel utilization, and the normalized effective throughput.

2 Networking Protocol (Roll Call)

The channel access protocol used in Link-11 is based on a centralized network control architecture, and it uses a single medium communication channel common to all nodes. The protocol assumes that the network is fully connected, i.e., each node can hear every other node. One of the nodes of the network is designated as the Data Net Control Station (DNCS), and controls the channel access of all other nodes in the network. All the other nodes are called picket stations, or Participating Units (PUs), and can only transmit information when prompted to do so by the DNCS. The automatic interrogation of the pickets, as described in reference [1], is summarized here.

The DNCS polls each picket station of the network in the order established by an address generator or by the Tactical Data System (TDS) computer. The interrogation message transmitted by the DNCS is composed of the following components:

a) Preamble and phase reference
b) Picket address

The picket station whose address was polled then transmits a reply message with the following components:

a) Preamble and phase reference
b) Start code
c) Any number of TDS message frames
d) Picket stop code

If a DNCS does not recognize a valid reply from a picket (i.e., if it does not recognize a start code) within 15 frames after interrogation, the DNCS shall send another interrogation to the same picket.

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If the DNCS does not recognize a valid reply to the second interrogation within 15 frames, it shall interrogate the next picket in the polling list.

If the DNCS receives a start code after either the first or second interrogation of a picket station, the DNCS will not interrogate the next picket until either of the following occurs:

a) A picket stop code is recognized
b) Loss of signal presence is determined

When the DNCS determines that it is next in line for transmitting TDS data (i.e., when the DNCS is the next node in the polling list) , it transmits a message whose frame structure includes both TDS data and an interrogation of the next node in the polling list. This frame structure consists of the following components:

a) Preamble and phase reference
b) Start code
c) Any number of TDS message frames
d) Stop code
e) Picket address of next picket to be interrogated

3 Performance Statistics

This section describes the performance statistics calculated in the model. The model provides estimates of what these statistics would actually be if the network were to operate under the stated conditions.

3.1 Net Cycle Time

The net cycle time as calculated in this model is defined as the total time required for the DNCS to interrogate all picket stations in the network, plus the reply times of the picket stations, plus the duration of the DNCS's own TDS transmission. In our model, the DNCS polls each picket station once each net cycle, with up to two interrogations per polling attempt. In non-ideal channel conditions, not all picket stations will inject TDS traffic in every net cycle. This is because message errors in the DNCS interrogations are introduced by the imperfect channel, thus preventing some pickets from hearing their interrogations. The DNCS may also not hear a picket reply even though a picket station has correctly responded to the DNCS's polling request. These imperfections in the channel conditions affect the net cycle time, and our model attempts to show how the network performs in these non-ideal conditions.

3.2 Total throughput Per Net Cycle

The total throughput per net cycle (denoted as R_{NC}) is defined as the sum of all the usable TDS bits received at each node in a net cycle. For bits to be usable by a node, they must be: 1) received without error by the node, 2) contained in M-series messages in which

the entire M-series message is received without error, and 3) contained in a data transmission in which the message indicator (MI) was also received without error. MI's are transmitted by every node before they transmit their TDS data so that all authorized receivers can decript the TDS data . For every M-series message transmitted in a net cycle, and for every node that receives the M-series message error-free, we add to R_{NC} the number of bits in an M-series message. Thus, if a message is correctly received by *n* nodes, it is added to R_{NC} *n* times.

3.3 Normalized Effective Throughput

The normalized effective throughput is defined in this model as the expected number of TDS bits successfully received per node in the network. The measure of the normalized effective throughput is in bits per second, and is normalized so that it is equivalent to the throughput of a network having a single transmitter and single receiver. We define the average normalized effective throughput as the average of the total error-free bits received per net cycle, divided by the product of the average net cycle time and the number of nodes in the network minus one.

$$\text{Ave. Normalized Effective Throughput} \equiv \frac{\text{ave. total throughput per net cycle}}{\text{(ave. net cycle time)(number of nodes - 1)}}$$

Bits contribute to the normalized effective throughput only if the entire message that contains them is correctly received. Note that messages received incorrectly are not retransmitted; instead, updates to these messages are automatically sent in future net cycles.

3.4 Percent Channel Utilization

The percent channel utilization performance statistic describes what percentage of the channel capacity is devoted to each of the four following categories: 1) TDS injection utilization, 2) guardband and preamble utilization (switching times, preambles, phase references, propagation delays, time-outs, and loss of signal time-outs), 3) header utilization (start codes, stop codes, and MI's), 4) net management utilization (address codes). The utilization in all four categories always sums to 100 percent.

4 The Model

Sections 4.1 and 4.2 of this paper lay down the foundations of our network model. They include a level of detail that will not be used in the other sections of this paper. However, we have included the detail in these sections in order to describe not only our model, but also the methodology of our approach, and its supporting theory. In this manner, we can use a similar approach to expand on this model in a more straightforward fashion, or to develop new models of other networks in which this approach is appropriate.

4.1 Model Description

The model is based on the DNCS state diagram shown in Figure 1. We divide the state diagram into a sequence of stages. Each stage starts when the DNCS interrogates a PU for the first time in the net cycle, and ends just before the DNCS interrogates the next PU. That is, a stage represents a polling transaction between the DNCS and a picket station. A network of N nodes will have N-1 stages (the DNCS does not interrogate itself). Each PU in the network is polled once per net cycle. A total of 4 true/false conditions are tested in this diagram in order to transition from one state to another. They are the following.

(a_1) DNCS correctly receives start code sent by PU after 1st interrogation attempt
(a_2) DNCS correctly receives start code sent by PU after 2nd interrogation attempt
(a_3) DNCS correctly receives stop code sent by PU after receiving start code
(a_4) DNCS determines loss of signal presence after receiving start code

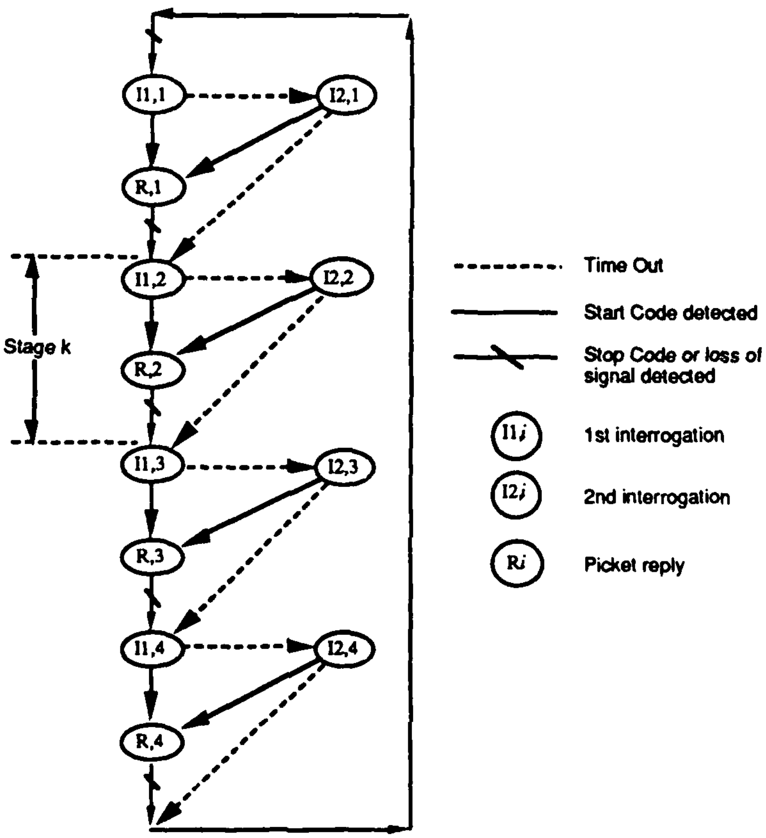


Figure 1. Roll-Call DNCS State Diagram (5 node network)

Since each test condition can only be true or false, we can define a total of 2^4 (i.e., 16) true/false combinations. Each of these 16 combinations (denoted by $c_0, c_1, \ldots, c_{15}$) is an element in the set of all possible outcomes of a stage. This set is the sample-space of a

stage in our model, and is denote as Ω_E. We let the event-space of a stage in our model, denoted it as $\mathcal{F}_E$, be the sigma-field generated by Ω_E. Thus, each of the 16 combinations is a possible event in a stage of our model. We denote the 16 events in $\mathcal{F}_E$ as the sets C_i, where we let $C_i = \{c_i\}$. However, only 5 of these 16 events have non-zero probabilities. Figure 2 is a table that shows which events have non-zero probability and which events have zero probability.

C_i	$= \{(a_1$	a_2	a_3	$a_4)\}$	$P(C_i) =$
C_0	0	0	0	0	b_5
C_1	0	0	0	1	0
C_2	0	0	1	0	0
C_3	0	0	1	1	0
C_4	0	1	0	0	0
C_5	0	1	0	1	b_4
C_6	0	1	1	0	b_3
C_7	0	1	1	1	0
C_8	1	0	0	0	0
C_9	1	0	0	1	b_2
C_{10}	1	0	1	0	b_1
C_{11}	1	0	1	1	0
C_{12}	1	1	0	0	0
C_{13}	1	1	0	1	0
C_{14}	1	1	1	0	0
C_{15}	1	1	1	1	0

(a "1" signifies that condition a_k is true, a "0" signifies that condition a_k is false)

Figure 2. Possible Events in a Stage

The reasons for setting some of the event probabilities to zero are as follows:

- The probabilities of events C_1, C_2, and C_3 are zero because we assume that the DNCS will not identify a start code and/or a loss of signal presence if it has not previously received a start code.

- The probabilities of events C_4 and C_8 are zero because if the DNCS receives a start code, it will always at some time determine either that it has received a stop code or determined a loss of signal presence condition. This is based on our assumption that the DNCS cannot falsely maintain modem synchronization if no node is transmitting real data.

- The probabilities of events C_3, C_7, C_{11}, and C_{15} are zero because the DNCS cannot both determine the loss of signal presence, and the reception of a stop code in the same stage. Recall that we have defined the stages as non-overlapping, and recall that either of the two sub-events a_3 and a_4 marks the end of a stage.

- The probabilities of events C_{12}, C_{13}, C_{14} and C_{15} are zero because, in these events, the DNCS has recognized that its first interrogation attempt was successful, and it will therefore not engage in a second interrogation attempt in the same stage. If the DNCS receives a start code, it will not attempt to decode another one until the next stage.

Throughout the rest of this paper, we will refer to the sets C_i that have probabilities greater than zero as events 1 through 5. We will refer to these sets as follows:

C_{10} = event 1		$P[C_{10}] = b_1$
C_9 = event 2		$P[C_9] = b_2$
C_6 = event 3		$P[C_6] = b_3$
C_5 = event 4		$P[C_5] = b_4$
C_0 = event 5		$P[C_0] = b_5$

Note that these events are mutually exclusive, and that the sum of their probabilities, $b_1 + b_2 + ... + b_5$, must equal one. A total of five distinct events can occur (i.e., have non-zero probabilities) when transitioning from one stage to the next. We have ignored all other events because their probabilities equal zero. The model assumes that the network is always in one of the N-1 stages, and only one of the 5 events can occur at a time within a stage.

The range of applications for this model is somewhat limited, however, because we have assumed that *no collisions in transmissions occur*. That is, the transmission of a node can never interfere with the transmission of another node. Thus, we assume that two or more nodes never transmit at the same time. Our assumption of no collisions requires that once the DNCS detects signal presence from a PU's transmission (i.e., it has synchronized with the PU's transmission and has correctly received a start code), the DNCS will always detect signal presence throughout the remainder of the PU's transmission. This is a reasonable assumption as long we assume that fading in the channel is low. Note that signal presence can be maintained even when the actual contents of a transmission are not decipherable. The detection of colliding transmissions is not a test condition defined by the Link-11 protocol. We therefore did not address this issue at this time in order to limit the complexity of our model. At minimum, we would have to develop a more complex state diagram, and define a larger set of possible events in order to accommodate collisions. An interpretation of the 5 events that have probabilities greater than zero is presented here.

Event 1 (C_{10}): The DNCS begins by interrogating a PU. The picket station correctly receives the first interrogation command from the DNCS, and responds to it appropriately. The DNCS correctly receives the start and stop codes of the PU's reply.

Event 2 (C_9): The DNCS begins by interrogating a PU. The PU correctly receives the first interrogation command from the DNCS, and responds to it appropriately. The DNCS correctly receives the PU's start code, but does not correctly receive the PU's stop code. After a time-out period following the end of

the PU's transmission, the DNCS determines a loss of signal presence, and proceeds to the next stage. The effect of not correctly receiving the stop code is the addition of a time-out period to the duration of event 1.

Event 3 (C_6): The DNCS begins by interrogating a PU. The PU does not correctly receive the first interrogation command, but does correctly receive the second interrogation command. The PU appropriately responds to the second interrogation command. The DNCS correctly receives the start and stop codes of the PU's reply.

Event 4 (C_5): The DNCS begins by interrogating a PU. The PU does not correctly receive the first interrogation command, but does correctly receive the second interrogation command. The PU appropriately responds to the second interrogation command. The DNCS correctly receives the PU's start code, but does not correctly receive the PU's stop code. After a time-out period following the end of the PU's transmission, the DNCS determines a loss of signal presence, and proceeds to the next stage. The effect of not correctly receiving the stop code is the addition of a time-out period to the duration of event 3.

Event 5 (C_0): The DNCS begins by interrogating a picket station. The picket station does not hear the first interrogation command and does not hear the second interrogation command. The PU therefore does not transmit any messages. The DNCS proceeds to the next stage.

We now evaluate the probability of these five elementary events. In any given stage, we define P_E[Event i] as the probability that the ith event occurs in that stage. The probabilities of the events are the following:

P_E[Event 1] = b_1 = (Prob. of detecting the start code)
x (Prob. of detecting the stop code)

P_E[Event 2] = b_2 = (Prob. of detecting the start code)
x [1.0 - (Prob. of detecting the stop code)]

P_E[Event 3] = b_3 = [1.0 - (Prob. of detecting the start code)]
x (Prob. of detecting the start code)
x (Prob. of detecting the stop code)

P_E[Event 4] = b_4 = [1.0 - (Prob. of detecting the start code)]
x (Prob. of detecting the start code)
x [1.0 - (Prob. of detecting the stop code)]

P_E[Event 5] = b_5 = [1.0 - (Prob. of detecting the start code)]
x [1.0 - (Prob. of detecting the start code)]

Note that $b_1 + b_2 + ... + b_5$ equals one, which is necessary for a consistent description of the probabilities and the probability measure P_E. The probabilities in the expressions above are calculated as follows:

(Prob. of detecting the start code) $= p_{synch1} \times p_{address} \times p_{synch2} \times p_{start}$

(Prob. of detecting the stop code) $= p_{stop}$

where

p_{synch1} = Pr(PU achieved synchronization with the DNCS transmission via the preamble and phase sent by DNCS)

$p_{address}$ = Pr(PU correctly decoded its address sent by DNCS given that the PU is already synchronized with the DNCS)

p_{synch2} = Pr(DNCS achieved synchronization with the PU transmission via the preamble and phase sent by PU)

p_{start} = Pr(DNCS correctly received start code sent by PU given that the DNCS is already synchronized with the PU)

p_{stop} = Pr(DNCS correctly received stop code sent by PU given that the DNCS is already synchronized with the PU)

The probability that the DNCS detects loss of signal presence is [1.0 - (Prob. of detecting the stop code)]. That is, once the DNCS has achieved synchronization and has correctly received a start code from a PU, it may determine a loss of signal presence only if it has not correctly received the stop code at the end of the PU's transmission. Signal presence loss is thus only detected if no signal exists. If the probability of detecting the stop code is 1, then the probability of determining a loss of signal presence is 0. The DNCS can detect signal presence even if it cannot correctly decode the messages in a transmission. Again, this assumption is needed in order to avoid collisions between transmissions.

4.2 Derivation of Net Cycle Time (General Form)

Before continuing, we define some parameters, constants, and random variables that we use in our analysis.

Parameters and Constants:

N = number of nodes in the network
$M_{preamble}$ = preamble duration
M_{phase} = phase duration
$M_{address}$ = address duration
M_{start} = start code duration

M_{stop} = stop code duration
M_{to} = time-out duration
$M_{switchRT}$ = receive to transmit switching duration
$M_{switchTR}$ = transmit to receive switching duration
M_{prop} = propagation delay duration
M_{loss} = signal presence loss determination duration
M_{MI} = message indicator duration for the crypto
M_{synch} = $M_{preamble} + M_{phase}$ = synchronization duration
$M_{polling}$ = $M_{switchRT} + M_{synch} + M_{address}$ = DNCS polling duration
M_{T_1} = constant time duration associated with event 1
M_{T_2} = constant time duration associated with event 2
M_{T_3} = constant time duration associated with event 3
M_{T_4} = constant time duration associated with event 4
M_{T_5} = constant time duration associated with event 5

Random Variables:

$M_{\text{message}}(\omega)$ = message duration at a node in a stage.

$M_i(\omega_i)$ = message duration in stage i of a net cycle (also denoted as M_i)

$T_i(\omega)$ = duration of event i in a stage (also denoted as T_i)

$D_{ST}(\omega,c)$ = duration of a stage (also denoted as D_{ST})

$D_i(\omega_i,c_i)$ = duration of stage i in a net cycle (also denoted as D_i)

$D_{NC}(\boldsymbol{\omega},\mathbf{c})$ = duration of a net cycle (also denoted as D_{NC}).

(Note that $\boldsymbol{\omega}$ and $\mathbf{c}$ are vectors $[\omega_{(1)},\omega_{(2)},\ldots,\omega_{(N-1)}]$ and $[c_{(1)},c_{(2)},\ldots,c_{(N-1)}]$ respectively, and have elements that are indexed by the stages in a net cycle.)

The random variable $M_{\text{message}}(\omega)$ represents the duration of the TDS data transmission at a PU. It does not include the duration of the message indicator (MI), which has already been defined in the above list of constants. $M_{\text{message}}(\omega)$ is defined over the probability space $(\Omega_M,\sigma(\Omega_M), P_M)$, where the elements of Ω_M represent the number of M-series messages generated at a node in a stage, and where P_M is the probability measure of $M_{\text{message}}(\omega)$. We will define P_M in more detail later in this paper.

The random variable $T_i(\omega)$ represents the duration of event i in a stage. Each $T_i(\omega)$ is a function of $M_{\text{message}}(\omega)$, and is defined over the same probability space as $M_{\text{message}}(\omega)$. We define a separate $T_i(\omega)$ for each of the five events that have non-zero probabilities. The $T_i(\omega)$'s are calculated as follows:

$$\begin{aligned} T_1(\omega) &= M_{polling} + M_{prop} + M_{switchRT} + M_{synch} + M_{start} + M_{MI} \\ &\quad + M_{\text{message}}(\omega) + M_{stop} + M_{prop} \\ &= M_{\text{message}}(\omega) + M_{T_1} \end{aligned}$$

$$
\begin{aligned}
T_2(\omega) &= \mathrm{M_{polling}} + \mathrm{M_{prop}} + \mathrm{M_{switchRT}} + \mathrm{M_{synch}} + \mathrm{M_{start}} + \mathrm{M_{MI}} \\
&\quad + M_{\mathrm{message}}(\omega) + \mathrm{M_{stop}} + \mathrm{M_{prop}} + \mathrm{M_{loss}} \\
&= M_{\mathrm{message}}(\omega) + \mathrm{M}_{T_2}
\end{aligned}
$$

$$
\begin{aligned}
T_3(\omega) &= \mathrm{M_{polling}} + \mathrm{M_{to}} + \mathrm{M_{polling}} + \mathrm{M_{prop}} + \mathrm{M_{switchRT}} + \mathrm{M_{synch}} \\
&\quad + \mathrm{M_{start}} + \mathrm{M_{MI}} + M_{\mathrm{message}}(\omega) + \mathrm{M_{stop}} + \mathrm{M_{prop}} \\
&= M_{\mathrm{message}}(\omega) + \mathrm{M}_{T_3}
\end{aligned}
$$

$$
\begin{aligned}
T_4(\omega) &= \mathrm{M_{polling}} + \mathrm{M_{to}} + \mathrm{M_{polling}} + \mathrm{M_{prop}} + \mathrm{M_{switchRT}} + \mathrm{M_{synch}} \\
&\quad + \mathrm{M_{start}} + \mathrm{M_{MI}} + M_{\mathrm{message}}(\omega) + \mathrm{M_{stop}} + \mathrm{M_{prop}} + \mathrm{M_{loss}} \\
&= M_{\mathrm{message}}(\omega) + \mathrm{M}_{T_4}
\end{aligned}
$$

$$
\begin{aligned}
T_5(\omega) &= \mathrm{M_{polling}} + \mathrm{M_{to}} + \mathrm{M_{polling}} + \mathrm{M_{to}} \\
&= \mathrm{M}_{T_5}
\end{aligned}
$$

The constants M_{T_i} represent the sum of the constants associated with each event i.

The random variable $D_{\mathrm{ST}}(\omega,\mathrm{c})$ represents the duration of a stage. Note that $D_{\mathrm{ST}}(\omega,\mathrm{c})$ is a function of both ω and c. That is, $D_{\mathrm{ST}}(\omega,\mathrm{c})$ is a function of the number of messages that a node must transmit, and a function of the five events that have non-zero probabilitie. $D_{\mathrm{ST}}(\omega,\mathrm{c})$ is a function over the probability space $(\Omega_{\mathrm{ST}},\sigma(\Omega_{\mathrm{ST}}), \mathrm{P}_{D_{\mathrm{ST}}})$, where $\Omega_{\mathrm{ST}} = \Omega_{\mathrm{E}} \times \Omega_M$, and where $\mathrm{P}_{D_{\mathrm{ST}}} = \mathrm{P_E}\mathrm{P}_M$. The duration of a stage is then calculated as

$$
D_{\mathrm{ST}}(\omega,\mathrm{c}) = \begin{cases}
T_1(\omega) & \text{if } \mathrm{c} = \mathrm{c}_0 \\
T_2(\omega) & \text{if } \mathrm{c} = \mathrm{c}_5 \\
T_3(\omega) & \text{if } \mathrm{c} = \mathrm{c}_6 \\
T_4(\omega) & \text{if } \mathrm{c} = \mathrm{c}_9 \\
T_5(\omega) & \text{if } \mathrm{c} = \mathrm{c}_{10} \\
0 & \text{otherwise (i.e., all other } \mathrm{c}_i\text{'s)}
\end{cases}
$$

Note that our event space Ω_{ST} is much larger than the one we first defined. An event in a stage of our model is no longer just one of the sixteen possible c_i's, but is one of the c_i's with an associated message length taken from ω. The expectation for $D_{\mathrm{ST}}(\omega,\mathrm{c})$ is calculated as follows:

$$
\begin{aligned}
\mathrm{E}[D_{\mathrm{ST}}(\omega,\mathrm{c})] &= \sum_{i=0}^{15} \left[\int_{\Omega_{\mathrm{M}}} \mathrm{T}_i(\omega)\, d\mathrm{P_M}(\omega) \right] \mathrm{P_E}[\mathrm{C}_i] \\
&= \sum_{i=1}^{5} \left[\int_{\Omega_M} T_i(\omega)\, d\mathrm{P}_M(\omega) \right] b_i
\end{aligned}
$$

$$= \sum_{i=1}^{5} E[T_i(\omega)]\ b_i$$

The random variable $T_i(\omega)$ could be either continuous or discrete. In our model however, we let $T_i(\omega)$ be a discrete random variable because we define $M_{\text{message}}(\omega)$ as a discrete random variable. The second moment of D_{ST} is expressed as follows:

$$E[D_{ST}^2] = \sum_{i=1}^{5} E[T_i^2(\omega)]\ b_i$$

Given the first and second moments of D_{ST}, we calculate the variance of D_{ST} as

$$\begin{aligned} \mathrm{Var}(D_{ST}) &= E[D_{ST}^2] - E[D_{ST}]^2 \\ &= \sum_{i=1}^{5} E[T_i^2(\omega)]\ b_i - \mu_{D_{ST}}^2 \end{aligned}$$

where $\mu_{D_{ST}}$ is the expectation of D_{ST}.

The random variable D_{NC} represents the duration of a net cycle, and is the sum of the duration of each stage, plus the duration of one start code, one stop code, and the duration of the TDS data transmitted by the DNCS in a net cycle. The last three terms are necessary because the stages, as we have defined them in our model, do not include the duration of the TDS data transmitted by the DNCS. We express D_{NC} for an N node network as follows:

$$\begin{aligned} D_{NC} &= D_1 + D_2 + D_3 + \ldots + D_{N-1} + M_{\text{start}} + M_{\text{MI}} + M_N + M_{\text{stop}} \\ &= \sum_{k=1}^{N-1} D_k + M_N + M_{\text{start}} + M_{\text{MI}} + M_{\text{stop}} \end{aligned}$$

M_N is the TDS message duration of the DNCS. Note that the components of D_{NC} are always assumed to be independent random variables. That is, the duration of one stage is independent of the duration of another stage. Thus, the duration of a TDS transmission at one node is independent of the duration of a TDS transmission at another node. However, the D_k's are not necessarily identically distributed. The traffic load is not necessarily the same for all nodes; different nodes may have different numbers of tracks. The probabilities of occurrence of the five elementary events may also differ for each of the N-1 stages. The probabilities of the events may differ due to differences in the link conditions between the PUs and the DNCS. The expected value of D_{NC} can be written as

$$\begin{aligned} E[D_{NC}] &= E[D_1] + E[D_2] + \ldots + E[D_{N-1}] + E[M_N] + M_{\text{start}} + M_{\text{MI}} + M_{\text{stop}} \\ &= \sum_{k=1}^{N-1} E[D_k] + E[M_N] + M_{\text{start}} + M_{\text{MI}} + M_{\text{stop}} \end{aligned}$$

Since the variance of a sum of independent random variables is equal to the sum of the variances of the random variables, we have

$$\mathrm{Var}(D_{\mathrm{NC}}) = \sum_{k=1}^{N-1} \mathrm{Var}(D_k) + \mathrm{Var}(M_N)$$

If the model were expanded to include collisions, the components of D_{NC} would not be independent. The duration of a stage could be dependent on what has happened in other stages, and our expression for the variance of D_{NC} would not be valid.

4.3 Derivation of Simplified Expressions for the Net Cycle Time

In this section, we derive a simplified expression of the net cycle time. We assume that each track at a node generates either one or two M-series messages per net cycle period (in the case of an air track, the probabilities of generating one or two messages are both equal to one-half). In the current Link-11 system, the duration of one M-series message is two modem frames, each frame containing 30 bits. This includes 48 data bits and 12 EDAC bits per M-series message. Given that m M-series messages are generated at a node, the duration of the TDS portion of a node's transmission is then

$$M_{\mathrm{TDS}}(m) = m\left(2\,\frac{\text{frames}}{\text{M-series}}\right)\left(0.01333\,\frac{\text{seconds}}{\text{frames}}\right) = \phi m$$

where $\phi = \left(2\,\frac{\text{frames}}{\text{M-series}}\right)\left(0.01333\,\frac{\text{seconds}}{\text{frames}}\right)$ represents the duration of the transmission of one M-series message. We let m equal the random variable $M_{(n)}(\omega)$, where

$$M_{(n)}(\omega) = n + Y_{(n)}(\omega).$$

$M_{(n)}(\omega)$ represents the number of M-series messages generated at a node in one stage. $Y_{(n)}(\omega)$ represents the number of tracks that generate two M-series messages instead of one. $Y_{(n)}(\omega)$ has a binomial distribution with $\mu = np_{\mathrm{M}}$, where n is the number of tracks at a node, and p_{M} is the probability that a track generates two M-series messages instead of one. The value of $Y_{(n)}(\omega)$ ranges from 0 to n, and the value of $M_{(n)}(\omega)$ ranges from n to $2n$. The random variable that describes the message duration at a node with n tracks is

$$M_{\mathrm{message}(n)}(\omega) = \phi[n + Y_{(n)}(\omega)]$$

To approximate the case of air tracks, we let p_{M} equal 0.5. The expected value of $M_{\mathrm{message}(n)}$ is

$$\begin{aligned} \mathrm{E}[M_{\mathrm{message}(n)}] &= \phi[n + np_{\mathrm{M}}] \\ &= \phi n\,(1+p_{\mathrm{M}}) \\ &= 1.5\phi n \qquad \text{for } p_{\mathrm{M}} = 0.5 \end{aligned}$$

The second moment of $M_{\text{message}(n)}(\omega)$ is

$$
\begin{aligned}
E\left[M^2_{\text{message}(n)}(\omega)\right] &= E\left[(\phi n + \phi\, Y_{(n)}(\omega))^2\right] \\
&= E\left[(\phi n)^2\right] + E\left[2\phi^2 n\ Y_{(n)}(\omega)\right] + E\left[\phi^2\, Y^2_{(n)}(\omega)\right] \\
&= \phi^2 n^2 + 2\phi^2 n\, p_M n + \phi^2\left[p_M n + p_M{}^2 n\,(n-1)\right] \\
&= \phi^2 n\,(p_M - p_M{}^2) + \phi^2 n^2 (1 + 2p_M + p_M{}^2) \\
&= 0.25\phi^2 n + 2.25\phi^2 n^2 \qquad \text{for } p_M = 0.5
\end{aligned}
$$

The variance of $M_{\text{message}(n)}$ is

$$
\begin{aligned}
\text{Var}[M_{\text{message}(n)}] &= \phi \text{Var}[Y_{(n)}] \\
&= \phi n p_M(1-p_M) \\
&= 0.25\phi n \qquad \text{for } p_M = 0.5
\end{aligned}
$$

Since we have now fully described $M_{\text{message}(n)}$, we can go ahead and describe the expectation and variance of the duration of event i in a stage having n tracks. The expectation of the duration of event i (i.e., the expectation of T_i) is calculated as follows:

$$
\begin{aligned}
E[T_i(\omega)] &= \begin{cases} E[M_{\text{message}(n)}(\omega)] + M_{T_i} & \text{for } i = 1,2,3,4 \\ M_{T_i} & \text{for } i = 5 \end{cases} \\
&= \begin{cases} \phi n\ (1+p_M) + M_{T_i} & \text{for } i = 1,2,3,4 \\ M_{T_i} & \text{for } i = 5 \end{cases}
\end{aligned}
$$

The second moment of $T_i(\omega)$, for $i = 1,2,3$ and 4, is calculated as follows.

$$
\begin{aligned}
E[T_i^2(\omega)] &= E\left[(M_{\text{message}(n)}(\omega) + M_{T_i})^2\right] \\
&= E\left[M^2_{\text{message}(n)}(\omega)\right] + 2\ M_{T_i}\ E\left[M_{\text{message}(n)}(\omega)\right] + M^2_{T_i} \\
&= \left[\phi^2 n\,(p_M - p_M{}^2) + \phi^2 n^2 (1+2p_M+p_M{}^2)\right] + 2\phi n\, M_{T_i}\,(1+p_M) + M^2_{T_i} \\
&= 0.25\phi^2 n + 2.25\phi^2 n^2 + 3\phi n\, M_{T_i} + M^2_{T_i} \qquad \text{for } p_M = 0.5
\end{aligned}
$$

And the second moment of $T_i(\omega)$ for $i = 5$ is

$$
E[T_5^2(\omega)] = M^2_{T_5}
$$

The variance of event i is

$$\mathrm{Var}[T_i(\omega)] = \begin{cases} \mathrm{Var}[M_{\mathrm{message}(n)}] = np_M(1-p_M) & \text{for } i = 1,2,3,4 \\ 0 & \text{for } i = 5 \end{cases}$$

All of the above expectations are with respect to n, the number of tracks at the node in the stage. For any given n and p_M, we can find a numerical answer for the expectations. Now that we know the statistics of $T_i(\omega)$, we can calculate the statistics of the duration of a stage, D_{ST}. The expectation and variance of D_{ST} are

$$\begin{aligned}
\mathrm{E}[D_{ST}] &= \sum_{i=1}^{5} \mathrm{E}[T_i(\omega)]\, b_i \\
&= \sum_{i=1}^{4} \left[\phi n\,(1+p_M) + M_{T_i}\right](b_i) + (M_{T_5})(b_5) \\
&= \left[\phi n\,(1+p_M) + M_{T_1}\right](b_1) \\
&\quad + \left[\phi n\,(1+p_M) + M_{T_2}\right](b_2) \\
&\quad + \left[\phi n\,(1+p_M) + M_{T_3}\right](b_3) \\
&\quad + \left[\phi n\,(1+p_M) + M_{T_4}\right](b_4) \\
&\quad + (M_{T_5})(b_5)
\end{aligned}$$

$$\begin{aligned}
\mathrm{E}[D_{ST}^2] &= \sum_{i=1}^{5} \mathrm{E}[T_i^2(\omega)]\, b_i \\
&= \left[\phi^2 n\,(p_M - p_M^2) + \phi^2 n^2 (1 + 2p_M + p_M^2)\right](b_1 + b_2 + b_3 + b_4) \\
&\quad + \left[2\phi n\, M_{T_1}(1+p_M) + M_{T_1}^2\right](b_1) \\
&\quad + \left[2\phi n\, M_{T_2}(1+p_M) + M_{T_2}^2\right](b_2) \\
&\quad + \left[2\phi n\, M_{T_3}(1+p_M) + M_{T_3}^2\right](b_3) \\
&\quad + \left[2\phi n\, M_{T_4}(1+p_M) + M_{T_4}^2\right](b_4) \\
&\quad + (M_{T_5}^2)(b_5)
\end{aligned}$$

$$\mathrm{Var}(D_{ST}) = \mathrm{E}[D_{ST}^2] - \mathrm{E}[D_{ST}]^2$$

(for p_M=0.5)

$$\begin{aligned}
= &\Big[(0.25\phi^2 n + 2.25\phi^2 n^2)(b_1 + b_2 + b_3 + b_4) + (3\phi n\, M_{T_1} + M_{T_1}^2)(b_1) \\
&+ (3\phi n\, M_{T_2} + M_{T_2}^2)(b_2) + (3\phi n\, M_{T_3} + M_{T_3}^2)(b_3) + (3\phi n\, M_{T_4} + M_{T_4}^2)(b_4) + (M_{T_5}^2)(b_5)\Big] \\
&- \Big[(1.5\phi n + M_{T_1})(b_1) + (1.5\phi n + M_{T_2})(b_2) + (1.5\phi n + M_{T_3})(b_3) \\
&+ (1.5\phi n + M_{T_4})(b_4) + (M_{T_5})(b_5)\Big]^2
\end{aligned}$$

We assume that the b_i's are the same for all stages and that all nodes have the same traffic load of n tracks. Because of this, we can let $M_k = M_{ST}$ for all k, and $D_k = D_{ST}$ for all k. We can then simplify the expression for $E[D_{NC}]$ and $Var[D_{NC}]$, and add the subscript (n) to indicate that each node has the same traffic load of n tracks.

$$\begin{aligned}
E[D_{NC(n)}] &= \sum_{k=1}^{N-1} E[D_k] + E[M_N] + M_{start} + M_{MI} + M_{stop} \\
&= (N-1)\, E[D_{ST}] + E[M_{ST}] + M_{start} + M_{MI} + M_{stop} \\
&= (N-1)\left[\sum_{i=1}^{5} E[T_i(\omega)]\, b_i\right] + 1.5\phi n + M_{start} + M_{MI} + M_{stop}
\end{aligned}$$

$$\begin{aligned}
(\text{for } p_M = 0.5) \quad &= (N-1)(1.5\phi n + M_{T_1})(b_1) \\
&\quad + (N-1)(1.5\phi n + M_{T_2})(b_2) \\
&\quad + (N-1)(1.5\phi n + M_{T_3})(b_3) \\
&\quad + (N-1)(1.5\phi n + M_{T_4})(b_4) \\
&\quad + (N-1)(M_{T_5})(b_5) \\
&\quad + 1.5\phi n + M_{start} + M_{MI} + M_{stop}
\end{aligned}$$

The variance can be expressed as

$$\begin{aligned}
Var(D_{NC}) &= \sum_{k=1}^{N-1} Var[D_{ST}] + Var[M_{ST}] \\
&= (N-1)\, Var[D_{ST}] + Var[M_{ST}]
\end{aligned}$$

(for p_M=0.5)

$$\begin{aligned}
= \; & (N-1)\Big[(0.25\phi^2 n + 2.25\phi^2 n^2)(b_1 + b_2 + b_3 + b_4) + (3\phi n\, M_{T_1} + M_{T_1}^2)(b_1) \\
& +(3\phi n\, M_{T_2} + M_{T_2}^2)(b_2) + (3\phi n\, M_{T_3} + M_{T_3}^2)(b_3) + (3\phi n\, M_{T_4} + M_{T_4}^2)(b_4) + (M_{T_5}^2)(b_5)\Big] \\
& - (N-1)\Big[(1.5\phi n + M_{T_1})(b_1) + (1.5\phi n + M_{T_2})(b_2) + (1.5\phi n + M_{T_3})(b_3) \\
& + (1.5\phi n + M_{T_4})(b_4) + (M_{T_5})(b_5)\Big]^2 + 0.25\phi n
\end{aligned}$$

4.4 Derivation of Channel Utilization

Under the same simplifying assumption as the last section (i.e., uniform channel conditions and uniform track load), we can represent the TDS injected traffic duration (i.e., TDS injection duration) in one net cycle as

$$M_{TDS/NC(n)} = \sum_{i=1}^{N-1} M_{TDS/st(n)} + M_{message(n)}$$

where we define the TDS injection duration for one stage with n tracks as

$$M_{\mathrm{TDS/st}(n)} = \begin{cases} M_{\mathrm{message}(n)} & \text{if } c = c_{10}, c_9, c_6, c_5 \\ 0 & \text{if } c = c_0 \end{cases}$$

Note that n has the same value for all nodes because of our assumption of a uniform traffic load among nodes. The expectation of $M_{\mathrm{TDS/NC}(n)}$ is

$$\mathrm{E}[M_{\mathrm{TDS/NC}(n)}] = (N\text{-}1)\mathrm{E}[M_{\mathrm{message}(n)}](b_1 + b_2 + b_3 + b_4) + \mathrm{E}[M_{\mathrm{message}(n)}]$$

$$(\text{for } p_M = .5) \quad = 1.5\phi n\,(N\text{-}1)(b_1 + b_2 + b_3 + b_4) + 1.5\phi n$$

The total duration per net cycle devoted to the switching time, the preamble, the phase reference, propagation delays, the time-outs, and the loss of signal time-outs (i.e., guardband and preamble duration) can be represented as

$$M_{\mathrm{GB/Pre/NC}(n)} = \sum_{i=1}^{N\text{-}1} M_{\mathrm{GB/Pre/st}(n)}$$

where we define the guardband and preamble duration for one stage as

$$M_{\mathrm{GB/Pre/st}(n)} =$$

$$= \begin{cases} 2\,\mathrm{M_{switchRT}} + 2\,\mathrm{M_{preamble}} + 2\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}} & \text{if } c = c_{10} \\ 2\,\mathrm{M_{switchRT}} + 2\,\mathrm{M_{preamble}} + 2\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}} + \mathrm{M_{loss}} & \text{if } c = c_9 \\ 3\,\mathrm{M_{switchRT}} + 3\,\mathrm{M_{preamble}} + 3\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}} + \mathrm{M_{to}} & \text{if } c = c_6 \\ 3\,\mathrm{M_{switchRT}} + 3\,\mathrm{M_{preamble}} + 3\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}} + \mathrm{M_{to}} + \mathrm{M_{loss}} & \text{if } c = c_5 \\ 2\,\mathrm{M_{switchRT}} + 2\,\mathrm{M_{preamble}} + 2\,\mathrm{M_{phase}} + 2\,\mathrm{M_{to}} & \text{if } c = c_0 \end{cases}$$

The expectation of $M_{\mathrm{GB/Pre/NC}(n)}$ is

$$\begin{aligned} \mathrm{E}[M_{\mathrm{GB/Pre/NC}(n)}] = \\ & b_1\,(N - 1)(2\,\mathrm{M_{switchRT}} + 2\,\mathrm{M_{preamble}} + 2\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}}) \\ & + b_2\,(N - 1)(2\,\mathrm{M_{switchRT}} + 2\,\mathrm{M_{preamble}} + 2\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}} + \mathrm{M_{loss}}) \\ & + b_3\,(N - 1)(3\,\mathrm{M_{switchRT}} + 3\,\mathrm{M_{preamble}} + 3\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}} + \mathrm{M_{to}}) \\ & + b_4\,(N - 1)(3\,\mathrm{M_{switchRT}} + 3\,\mathrm{M_{preamble}} + 3\,\mathrm{M_{phase}} + 2\,\mathrm{M_{prop}} + \mathrm{M_{to}} + \mathrm{M_{loss}}) \\ & + b_5\,(N - 1)(2\,\mathrm{M_{switchRT}} + 2\,\mathrm{M_{preamble}} + 2\,\mathrm{M_{phase}} + 2\,\mathrm{M_{to}}) \ . \end{aligned}$$

The total duration per net cycle devoted to the start codes, stop codes, and the MI's (i.e., header duration), given the same simplifying assumptions as above, can be written as

$$M_{\mathrm{DB/header/NC}(n)} = \sum_{i=1}^{N\text{-}1} M_{\mathrm{DB/header/st}(n)} + \mathrm{M_{start}} + \mathrm{M_{MI}} + \mathrm{M_{stop}}$$

where we define the header duration for one stage as

$$M_{\text{DB/header/st}(n)} = \begin{cases} (M_{\text{start}} + M_{\text{MI}} + M_{\text{stop}}) & \text{if } c = c_{10}, c_9, c_6, c_5 \\ 0 & \text{if } c = c_0 \end{cases}$$

The expectation of $M_{\text{DB/header/NC}(n)}$ is

$$\begin{aligned} E[M_{\text{DB/header/NC}(n)}] = \; & (N\text{-}1)\,(M_{\text{start}} + M_{\text{MI}} + M_{\text{stop}})\,(b_1 + b_2 + b_3 + b_4) \\ & + M_{\text{start}} + M_{\text{MI}} + M_{\text{stop}} \end{aligned}$$

The total duration per net cycle devoted to the address codes (i.e., net management duration) can also be expressed as

$$M_{\text{Net/Mgmt/st}(n)} = \sum_{i=1}^{N-1} M_{\text{Net/Mgmt/st}(n)}$$

where we define the net management duration for one stage as

$$M_{\text{Net/Mgmt/st}(n)} = \begin{cases} M_{\text{address}} & \text{if } c = c_{10}, c_9 \\ 2M_{\text{address}} & \text{if } c = c_6, c_5, c_0 \end{cases}$$

The expectation of $M_{\text{Net/Mgmt/st}(n)}$ is

$$\begin{aligned} E[M_{\text{Net/Mgmt/NC}(n)}] = \; & (N\text{-}1)\,(M_{\text{address}})(b_1 + b_2) \\ & + (N\text{-}1)\,(2M_{\text{address}})(b_3 + b_4 + b_5) \end{aligned}$$

If the expected values that we have derived in this section are divided by the expected value of the net cycle time, $E[D_{\text{NC}(n)}]$, the result will yield the percent utilization of the channel capacity devoted to each of the respective categories.

4.5 Derivation of Normalized Effective Throughput

Let the random variable J represent the number of nodes that have correctly received the preamble, the phase reference, the start code, and the MI transmitted in a stage where the polling protocol succeeded between the DNCS and the picket station (events 1,2,3 or 4). By virtue of its definition, J represents the nodes that have the potential for contributing to the throughput statistics of a stage because these nodes have the potential for correctly receiving M-series messages in that stage. They have reached this status by having correctly received the picket station's transmission up to and including the MI. The value of J ranges from 0 to N-2 because we do not consider the transmitting picket station a candidate for receiving its own transmission, and we do not consider the DNCS because it comes under a different probability measure (i.e., we have already assumed that the DNCS has correctly received the preamble, phase reference and start code, so the DNCS only needs to correctly receive the MI in order for it to be a potential contributor to the throughput statistics). If we let p_J represent the probability that a PU has the potential for correctly receiving M-series messages in a stage, we can express it as follows.

$p_J = (p_{\text{synch}})\ (p_{\text{start}})\ (p_{\text{MI}})$.

Note that we assume that all errors are detected. The probability that j out of N-2 nodes have the potential for correctly receiving M-series messages is the probability that J equals j. Since the correct reception of a message at each node is independent of that at any other node, this probability can be expressed as

$$\Pr\{J=j\} = \binom{N-2}{j} p_J^{\,j}\,(1-p_J)^{N-2-j}$$

where the value of j ranges from 0 to N-2. From the above expressions, we can write the expectation of J as

$$\mathrm{E}[J] = (N-2)p_J$$

Suppose that one M-series message is transmitted in a given stage (i.e., events 1, 2, 3, or 4). We let the random variable X represent the number of nodes that correctly receive this M-series message. The domain of X ranges from 0 to the total number of nodes that are candidates for correctly receiving this M-series message (not considering the DNCS or the transmitting PU). We let p_s represent the probability that a node correctly receives an M-series message, assuming that it is a candidate for receiving the M-series message. If J is the total number of nodes that are candidates for correctly receiving the M-series message, then the probability that k out of J nodes receive this message is

$$\mathrm{P}\{X=k\} = \binom{J}{k} p_s^{\,k}\,(1-p_s)^{J-k}\ .$$

The value of k will range from 0 to J, and the value of J will range from 0 to N-2. We can express the expectation of X as

$$\begin{aligned}
\mathrm{E}[X] &= \sum_{i=1}^{N-2} \mathrm{E}\big[X \mid J=i\big]\ \Pr\{J=i\} \\
&= \sum_{i=1}^{N-2} [i\ p_s\,]\ \Pr\{J=i\} \\
&= p_s \sum_{i=1}^{N-2} i\ \Pr\{J=i\} \\
&= p_s\,\mathrm{E}[J] \\
&= p_s\,(N-2)p_J
\end{aligned}$$

E[X] is the expected number of nodes that will correctly receive an M-series message given that one M-series message was transmitted. We now define the random variable $R_{\text{M-series}_1}(k)$ as the total throughput of the transmission of one M-series message in an event having only one M-series message transmission. $R_{\text{M-series}_1}$ is defined as

$$R_{\text{M-series}_1}(k) = \mathrm{ß}\,X(k)$$

ß is the the number of TDS bits in an M-series message. If two M-series messages are transmitted, and $X_1(k_1)$ is the number of nodes that received the first message, and $X_2(k_2)$ is the number of nodes that received the second message, then the total throughput for the two messages is

$$R_{\text{M-series}_2}(\boldsymbol{k}) = ß[X_1(k_1) + X_2(k_2)]$$

The element $\boldsymbol{k}$ is now the ordered pair (k_1, k_2). If m M-series messages are transmitted, then the total throughput is

$$\begin{aligned} R_{\text{M-series}_m}(\boldsymbol{k}) &= ß[X_1(k_1) + X_2(k_2) + \ldots + X_m(k_m)] \\ &= ß \sum_{i=1}^{m} X_i(k_i) \quad . \end{aligned}$$

We assume that the reception of an M-series message at a node does not affect the reception of any other M-series messages, either at the same node, or at another node. We also assume that the the probability p_s, of correct reception of an M-series message at a node is a constant, and equal for all nodes. As a result of these assumptions, we have independent and identically distributed X_i's.

Here is a figurative summary of the computation of the random variable $R_{\text{M-series}_m}$. For each M-series message that is transmitted in a stage, we say that one "point" is "scored" by each node that correctly receives the M-series message. Only those nodes that have correctly received the MI for that event are allowed to "play" and score points. The number of "players" is J, and a new J is generated each event. The players have the opportunity to score one point each time a new M-series message is transmitted (that is, in each "round"). Each X_i stands for a separate round, and there are m rounds in each event. The "point value" of a score is ß and represents the number of bits contained in an M-series Message. $R_{\text{M-series}_m}$ is the total score from all players in a stage that is composed of m rounds. Thus, $R_{\text{M-series}_m}$ represents the total number of correctly received bits in an event in which m M-series messages were transmitted (excluding the bits that the DNCS may receive).

Recall, however, that the value of m is actually a random variable itself. We defined m earlier to be the random variable $M_{(n)}(\omega) = n + Y_{(n)}(\omega)$. Using $M_{(n)}(\omega)$ instead of m, we now define a new random variable, $R_{E_{(n)}}(\boldsymbol{k},\omega)$, that represents the *total throughput* in events 1, 2, 3, 4 or 5, with n tracks being reported on by the node in that event (and again, not counting the total throughput of the DNCS). The element $\boldsymbol{k}$ is now $(k_1, k_2, \ldots, k_n)$ and the value of $M_n(\omega)$ varies from n to $2n$. $R_{E_{(n)}}(\boldsymbol{k},\omega)$ is expressed as

$$R_{E_{(n)}}(\boldsymbol{k},\omega) = ß \sum_{i=1}^{M_{(n)}(\omega)} X_i(k_i)$$

We can find the expected value of $R_{E(n)}(\boldsymbol{k},\omega)$ as follows:

$$
\begin{aligned}
E[R_{E(n)}] &= \beta \sum_{i=n}^{2n} \Big[E[R_{E(n)} \mid M_{(n)} = i] \, \big(\text{Prob}[M_{(n)} = i]\big) \Big] \\
&= \beta \sum_{i=n}^{2n} \Big[\big(\sum_{j=1}^{i} E[X_j] \big) \, \big(\text{Prob}[M_{(n)} = i]\big) \Big] \\
&= \beta \sum_{i=n}^{2n} \Big[i \, \big(E[X]\big) \, \big(\text{Prob}[M_{(n)} = i]\big) \Big] \\
&= \beta \, E[X] \sum_{i=n}^{2n} \Big[i \, \big(\text{Prob}[M_{(n)} = i]\big) \Big] \\
&= \beta \, E[M_{(n)}] \, E[X] \\
&= \beta \, n p_s p_J (N\text{-}2)(1+p_M)
\end{aligned}
$$

Recall that p_M is the probability that a track generates two M-series messages instead of one.

We now define the random variable $R_{st(n)}(\boldsymbol{k},\omega,c)$ which represents the total throughput of a stage with n tracks being reported on by the node in the given stage. Note that this random variable does not count the total throughput of the DNCS. $R_{st(n)}(\boldsymbol{k},\omega,c)$ is expressed as follows:

$$
R_{st(n)}(\boldsymbol{k},\omega,c) = \begin{cases} R_{E(n)}(\boldsymbol{k},\omega) & \text{if } c = c_0, c_5, c_6, c_9 \\ 0 & \text{if } c = c_{10} \end{cases}
$$

The expectation of $R_{st(n)}(\boldsymbol{k},\omega,c)$ is

$$
\begin{aligned}
E[R_{st(n)}] &= \sum_{i=1}^{4} E[R_{E(n)}] \, b_i \\
&= E[R_{E(n)}] \sum_{i=1}^{4} b_i \\
&= (1\text{-}b_5) \, E[R_{E(n)}]
\end{aligned}
$$

So far, we have ignored the contribution of the DNCS to the total throughput (i.e., the expected number of messages successfully received by the DNCS). The DNCS was ignored because it already synchronized with the PU if events 1, 2, 3, or 4 occurred. The total throughput of the DNCS is therefore based only on the probability of correct MI and Message reception. We express the total throughput of the DNCS in a stage as

$$
R_{dncs(n)}(\omega,c) = \begin{cases} \beta R_n(\omega) & \text{if } c = c_0, c_5, c_6, c_9 \\ 0 & \text{if } c = c_{10} \end{cases}
$$

where $R_n(\omega)$ is the number of M-series messages received by the DNCS in either stages 1, 2, 3, or 4. $R_n(\omega)$ ranges from n to $2n$, and the probability that $R_n(\omega) = k$ is

$$P\{R_n{=}k\} = \quad p_{\mathrm{MI}} \binom{M_{(n)}}{k} p_s^{k} (1-p_s)^{M_n-k}$$

The average of the total throughput of a stage in which the DNCS transmits can be derived by using techniques similar to the ones we used earlier in this memorandum. This average can be expressed as the following:

$$\begin{aligned} E[R_{\mathrm{dncs}(n)}] &= \beta\, n p_s p_{\mathrm{MI}}(1+p_{\mathrm{M}})\,(b_1+b_2+b_3+b_4) \\ &= \beta\, n p_s p_{\mathrm{MI}}(1+p_{\mathrm{M}})\,(1-b_5) \end{aligned}$$

The total throughput in a stage ($R_{\mathrm{ST}(n)}$) is the sum of the total throughput of the picket stations, plus the total throughput of the DNCS. The expression for $R_{\mathrm{ST}(n)}(k,\omega)$ and its expected value is

$$\begin{aligned} R_{\mathrm{ST}(n)}(k,\omega) &= R_{\mathrm{st}(n)}(k,\omega) + R_{\mathrm{dncs}(n)}(k,\omega) \\ E[R_{\mathrm{ST}(n)}] &= E[R_{\mathrm{st}(n)}] + E[R_{\mathrm{dncs}(n)}] \\ &= \beta n p_s p_J(N-2)(1+p_{\mathrm{M}})(1-b_5) + \beta n p_s p_{\mathrm{MI}}(1+p_{\mathrm{M}})(1-b_5) \\ &= \beta\, n p_s (1+p_{\mathrm{M}})[p_J(N-2)+p_{\mathrm{MI}}]\,(1-b_5) \end{aligned}$$

The total throughput in a net cycle ($R_{\mathrm{NC}(n)}$) is the sum of the total throughput in N-1 stages ($R_{\mathrm{ST}(n)}$), plus the total throughput the PUs for the TDS data transmitted by the DNCS ($R_{\mathrm{DNCS}(n)}$), which is not accounted for in any of the stages. The expected value of $R_{\mathrm{DNCS}(n)}(k,\omega)$ is

$$E[R_{\mathrm{DNCS}(n)}] = \beta n p_s p_J(N-1)(1+p_{\mathrm{M}})$$

This value is almost the same as that of $E[R_{\mathrm{st}}(k,\omega)]$ except that the total number of possible receiving nodes is N-1 instead of N-2. This is because the DNCS is the transmitting node, and we, therefore, count all nodes except for the DNCS. If we let all nodes have the same number of tracks to report on, we can express the expected value of $R_{\mathrm{NC}(n)}(k,\omega)$ as:

$$\begin{aligned} E[R_{\mathrm{NC}(n)}] &= \sum_{i=1}^{N-1} E[R_{\mathrm{ST}(n)}] + E[R_{\mathrm{DNCS}(n)}] \\ &= (N-1)\, \beta n p_s (1+p_{\mathrm{M}})[p_J(N-2)+p_{\mathrm{MI}}](1-b_5) + \beta n p_s p_J(N-1)(1+p_{\mathrm{M}}) \\ &= \beta n p_s (N-1)(1+p_{\mathrm{M}}) \left\{ (1-b_5)[p_J(N-2)+p_{\mathrm{MI}}] + p_J \right\} \end{aligned}$$

Here again, we have assumed independence between stages. Using our earlier definition, if we divide the average total throughput of a net cycle by the average duration

of a net cycle (i.e., average net cycle time), we get the average TDS reception throughput of the network. This average is represented as

$$\mathrm{E}[H_{\mathrm{R}(n)}] = \frac{\mathrm{E}[R_{\mathrm{NC}(n)}]}{\mathrm{E}[D_{\mathrm{NC}(n)}]}$$

Furthermore, if we divide the average TDS reception throughput by N-1, the resulting value represents the average reception throughput normalized for a single channel. That is, it represents the effective TDS throughput of a channel in which TDS information is exchanged between a single transmitter and a single receiver. The normalized effective throughput is represented as

$$\mathrm{E}[H_{\mathrm{eff}(n)}] = \frac{\mathrm{E}[R_{\mathrm{NC}(n)}]}{(N-1)\mathrm{E}[D_{\mathrm{NC}(n)}]}$$

5 Results from the Model

This section provides some example results obtained from the probabilistic model. These types of results can be used in comparing the performance of different modems as they affect the performance of the network. Figure 3 shows the averages and standard deviations of the net cycle time for a parallel tone modem using the current Link-11 frame structure, when the probability of modem synchronization changes from zero to one. The values are calculated with network sizes of 10, 20, 30 and 40 nodes. The probabilities for correct reception of the picket addresses, the start codes, the stop codes, the MI's, and the M-series messages were set to one in generating figure 3. The traffic load on the network is 20 air tracks per node.

Note that the average net cycle time increases as the probability of synchronization increases, and that the increase is more pronounced with larger network sizes. This is due to the fact that more DNCS interrogations succeed as the probability of synchronization increases, resulting in an increased amount of TDS transmissions into the network. The average net cycle time decreases slightly when the probability of synchronization increases from about 0.9 to 1.0. This is because with a synchronization probability of 0.9, most of the picket stations correctly receive at least one of the two interrogations sent to them. With a synchronization probability of 1.0, all the nodes correctly receive the first interrogation, and the time-out period associated with the second interrogation is no longer present.

Plots of the averages and standard deviations of the net cycle times as functions of synchronization probability for a currently proposed single tone modem have also been generated. These plots are, as expected, virtually identical to the those of the parallel tone modem in figure 3. The reason for the differences is due to the fact that the data frame structure of the proposed single tone modem differs slightly from the frame structure of the current parallel modems. Currently, when a node transmits data (just after transmitting a start code), it transmits an MI (24 bits) followed by encrypted TDS data. The MI is used by the receiving nodes to decrypt the transmitted data. The proposed single tone modem

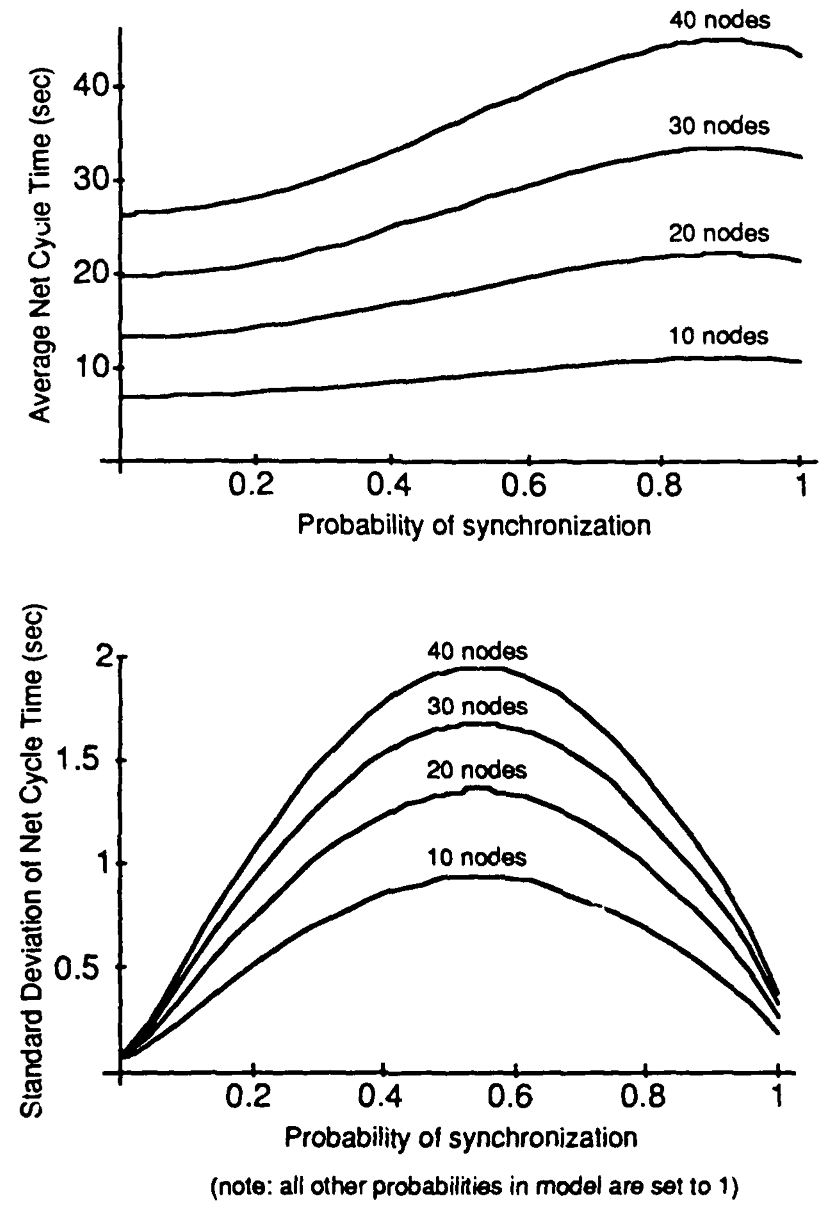


Figure 3. Average and Standard Deviation of Net Cycle Time vs. Probability of synchronization, with different network sizes (Parallel Tone Modem)

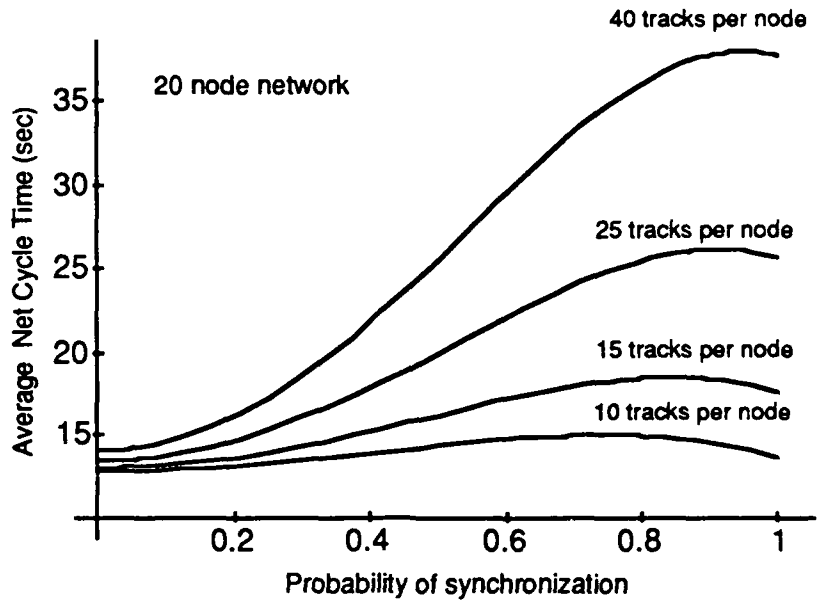
includes the MI in a special transmission header that is used as a substitute for the start code. The MI therefore not transmitted at the beginning of the TDS data since it is already transmitted in the header. We thus set the duration of the MI in our model to zero when modelling the proposed modem; all other event durations defined in the model are equal in both the current and proposed modems. This is because the effective synchronization duration, start code or header duration, and the effective user bit rates are the same for both modems. A discription of proposed modem' s frame structure is included in reference 2.

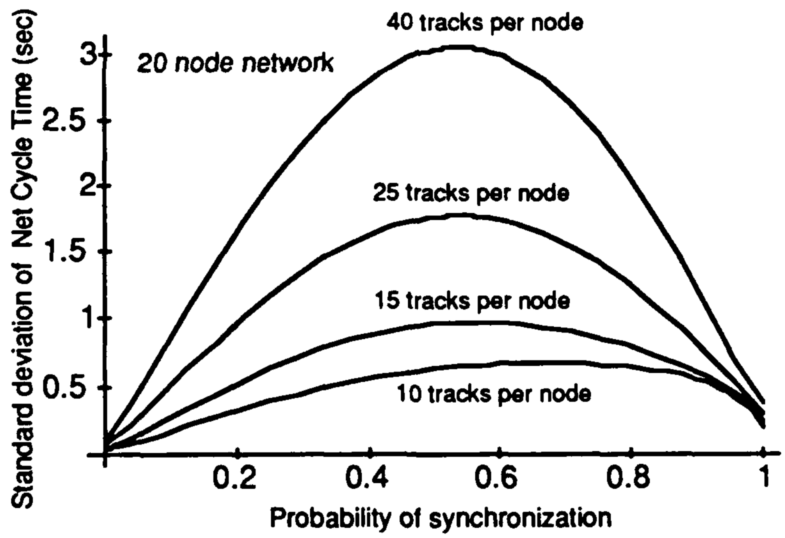
The major distinguishing differences between the current and proposed modems lie in the probabilities of synchronization and the bit error rates. The proposed modem uses error detection and correction coding techniques that were not available at the time of the design of the currently used modems. Under similar channel conditions, the probabilities of synchronization and of correct message reception are generally higher for the single tone modem than for the parallel tone modem. Since figure 3 applies to both modems, the operating points of the network for each modem will lie on different points of the graphs in figure 3 for any given set of channel conditions. These graphs can thus be used to compare the network perform with different modems under similar and uniform channel conditions.

Figure 4 shows the averages and standard deviations of the net cycle time under the same parameters as those used in Figure 3, except that the number of nodes in the network remains at 20, and the traffic load varies from 10, 15, 25, and 40 air tracks per node. When the probability of synchronization is low, the net cycle times are all very close for all four traffic loads. However, when the probability of synchronization is high, the average net cycle time is much higher with a heavy traffic load than with a light traffic load.

Figure 5 shows the average effective TDS throughput as the probability of synchronization changes from 0 to 1. A different plot is shown for traffic loads of 10, 15, 25, and 40 tracks per node. The probabilities for correct reception of the picket addresses, the start codes, the stop codes, the MI's, and the M-series messages were set to one in generating figure 5. The average throughput increases as the probability of synchronization increases. When the probability of synchronization is zero, the effective throughput is zero because none of the nodes ever hear the transmissions of the other nodes. Note that the throughput increases as the number of tracks at a node increases. This is because the portion of the channel devoted to TDS data increases relative to the network protocol overhead, and therefore, the TDS traffic occupies a larger percentage of the channel as the TDS message lengths increase. From Figure 4 however, we have seen that the average net cycle time also increases as the TDS message lengths increase.

Figure 6 shows the average effective TDS throughput as the probability of correctly receiving an M-series message changes from 0 to 1. A different plot is shown for traffic loads of 10, 15, 25, and 40 tracks per node. The probabilities of modem synchronization, of correct reception of the picket addresses, of the start codes, of the stop codes, and of the MI's were set to one. The results from Figure 6 are similar to those of Figure 5, except that the curves show a linear increase of average TDS effective throughput as the probability of correctly receiving an M-series message increases. This is a linear increase because the probability of correctly receiving an M-series message does not affect the net cycle time.





(note: all other probabilities in model are set to 1)

Figure 4. Average and Standard Deviation of Net Cycle Time with different traffic loads

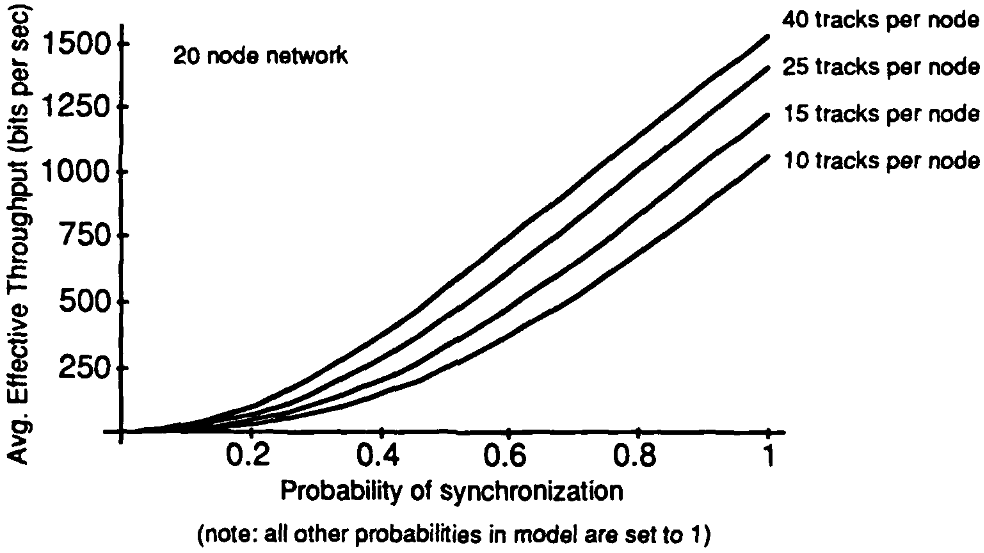


Figure 5. Effective Throughput vs. Probability of Synchronization with different traffic loads

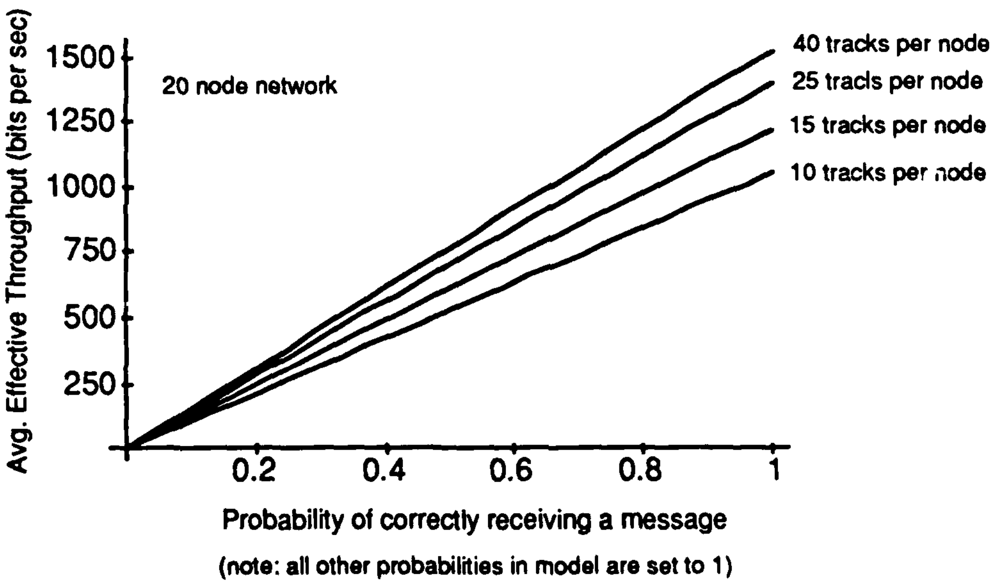


Figure 6. Effective Throughput vs. Probability of Correct Message Reception with different traffic loads

Figure 7 shows the effective throughput as the probability of correctly receiving the MI changes from 0 to 1. A different plot is shown for traffic loads of 10, 15, 25, and 40 tracks per node. The probabilities of synchronization, of correct picket address reception, of correct start-code reception, of correct stop-code reception, and of correct message reception were set to one. This figure is virtually identical to Figure 6. The reception of the MI does not affect the net cycle times. Note that even though all other probabilities are set to one, the throughput can still be poor if the probability of correctly receiving the MI is poor.

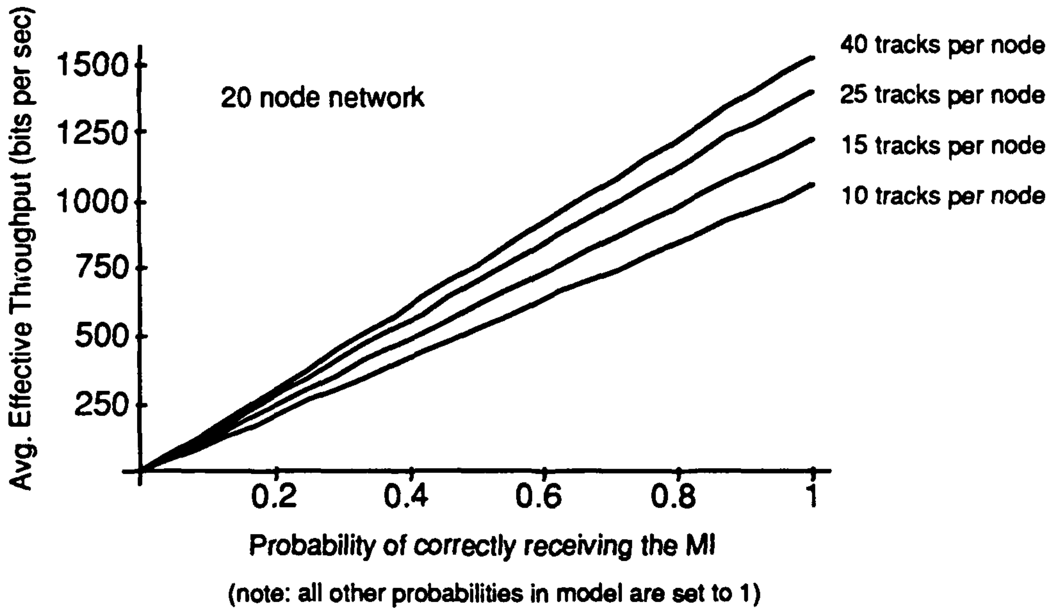
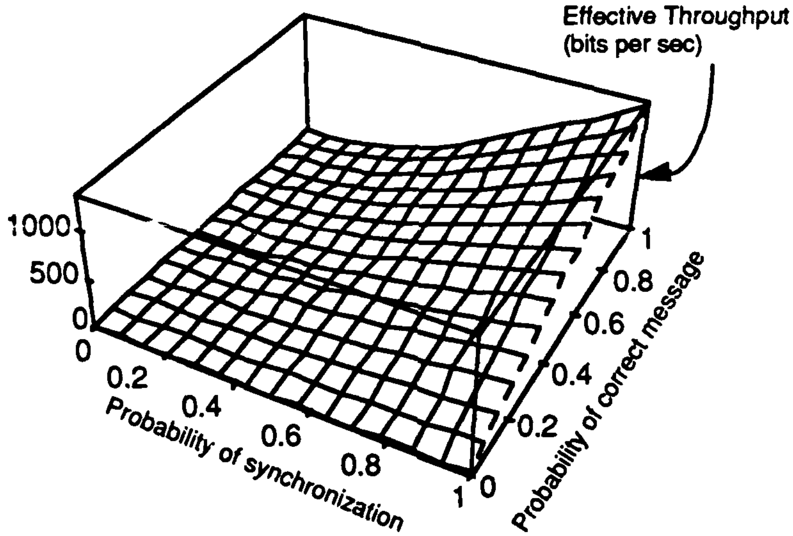


Figure 7. Effective Throughput vs. Probability of Correct MI Reception with different traffic loads

Figure 8 is a three dimensional plot of the average effective throughput as both the probability of synchronization and the probability of correct reception of M-series messages vary from 0 to 1. The probabilities of correct reception of picket addresses, the start codes, the stop codes, and the MI's were set to one. When either the probability of synchronization or the probability of correct message reception is zero, the effective throughput is also zero. Maximum throughput is achieved when all probabilities are one. The results of Figure 8 are also provided in table form in Figure 9. The values for the single tone modem vary from this table by only about 1 percent. The difference is due to the fact that the MI is included with the ILEI header.



(20 nodes, 20 tracks per node)

Note: the probability of correctly receiving the start code, the stop code, the MI, and the address are all 1 in this figure.

Figure 8. Effective Throughput for Parallel Tone Modem with different probabilities of synchronization and different probabilities of Correct Message Reception

Effective Throughput

for current parallel tone modem

Probability of Correct Message

Probability of Synchronization	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
0	0	0	0	0	0	0	0	0	0	0	0
0.1	0	2	3	5	7	8	10	12	13	15	17
0.2	0	6	11	17	23	28	34	40	46	51	57
0.3	0	13	26	40	53	66	79	93	106	119	132
0.4	0	24	49	73	98	122	147	171	196	220	244
0.5	0	39	78	116	155	194	233	271	310	349	388
0.6	0	55	111	166	222	277	333	388	444	499	554
0.7	0	74	147	221	295	369	442	516	590	663	737
0.8	0	93	186	279	372	465	558	651	744	837	930
0.9	0	113	226	339	451	564	677	790	903	1016	1128
1.0	0	133	266	399	532	665	798	931	1064	1197	1330

Results in table are in bits per second

20 node network, 20 tracks per node

All other probabilities in model were set to 1.

Figure 9. Effective Throughput for Parallel Tone Modem

6 Conclusions

We have presented in this paper an analytic model of the Link-11 communication network. The model gives us estimates of the average and standard deviation of the net cycle time, the average percent channel utilization, and the average effective throughput of the network. The model is based on some simplifying assumptions in order to limit its complexity and development time. As a result of the assumptions, the model is limited in its overall application to specific and detailed battle group scenarios. Care must be taken to understand the nature and significance of the assumptions, so that valid conclusions are drawn from its results.

One of the major assumptions in the model is that we ignore the effects of collisions in transmission. That is, we do not model the effects of two or more nodes transmitting at the same time, and thus interfering with each other's transmissions. In the actuality, nodes may well transmit at the same time. We point out that the Link-11 networking protocol is not designed to deal with collision conditions. As a result of collisions, some nodes may not be able to inject TDS traffic in every net cycle. One condition for collisions is when a picket station correctly receives its address from the DNCS, but the DNCS does not synchronize with the picket reply, or does not correctly receive the start code in the picket reply. Under these circumstances, the DNCS will transmit another interrogation message to the same PU, or a new interrogation message to another PU, resulting in a collision with the PU's reply message. These collisions can cause changes in the operation of the network, and can thus cause changes in the performance statistics of the network. The effects of the no-collision assumption become less important and less pronounced when the probabilities of synchronization and of correct start code receptions are high.

The second major assumption made in defining the model is that the channel conditions are uniform among all nodes of the network. In a real battle group deployment, the different relative distances between platforms and the atmospheric and oceanic conditions will actually result in non-uniform channel conditions, and thus result in different probabilities of the events in each of the stages in our model. We have assumed in our model that the probabilities of events within a stage are the same for all stages. Differences in actual event probabilities will affect the performance statistics of the network, depending on the deployment scenario of the battle group.

The third major assumption made in defining the model is that the traffic load is the same for all nodes in the network, and the statistical characteristics of the generation function of messages was simplified. In a real battle group deployment, different nodes will have different amounts and types of TDS traffic. We also assumed that no dependencies exist between TDS messages received at a node. That is, all TDS messages correctly received at a node are usable by that node. In actuality, some M-series messages types must be received without error in order for others to be usable by the receiving node. These message dependencies will affect the throughput of the network. Refinements to our model can be incorporated in the future to account for message dependencies.

The assumptions we have listed limit the number of battle group scenarios that can be represented by the model. More time and work would be required to eliminate these

assumptions from our model. We could also define new performance statistics such as track update rates or probability of transmission collisions. The model has a great potential for growth and refinement. In addition to the results generated from our model, the model has provided us with a methodology for modeling the Link-11 network in an analytic form. Using the same basic methodology presented in this paper, we could develop a more realistic model of the Link-11 network (albeit more complex), or develop models of other networks that have relatively simple networking protocols.

We believe that the model, as it currently stands, provides a useful means for comparing the effects of different modems as they affect the performance of the Link-11 network. Though the model assumes some idealistic conditions, one advantage of the model is that the comparisons of the performance statistics are on a network-wide level, and not just on a single link level. We can see how different modems affect the operation of the network as a whole. Another advantage of the model is that it provides numerical results, rather than just subjective impressions and predictions. These numerical results can form the basis for quantitative comparisons between different modem types, as they may perform in the Link-11 network. Under similar channel conditions, the probabilities of synchronization and of error free message reception for different modem types can be measured through field testing and/or the use of channel simulators. These test results can input directly into the model to estimate the network's effectiveness under different modems.

REFERENCES

[1] MIL-STD-188-203-1A 31 DECEMBER 1986 (DRAFT)

[2] Doc. No. J001BC0019-03, 26 July 89, "System Specification for the Multimedia Improved Link Eleven System," Rockwell/Marconi Joint Venture Team, Richardson TX; prepared for Space and Naval Warfare Systems Command, Washington, DC.